Since from the preceding corollary these equations are impossible, the truth of the theorem follows at once.

From theorems III and VII we have the following corollary:

COROLLARY. *The number expressing the area of a numerical right triangle has at least one odd prime factor entering to an odd power.*

Now a number of the form $rs(r^2 - s^2)$, where r and s are different positive integers, is the area of the numerical right triangle determined by the equation

$$(2rs)^2 + (r^2 - s^2)^2 = (r^2 + s^2)^2.$$

Put $r = \rho^2$ and $s = \sigma^2$. Then from the above corollary we see that $\rho^2\sigma^2(\rho^4 - \sigma^4)$ has some odd prime factor entering into it to an odd power. Since every prime factor of $\rho^2\sigma^2$ obviously enters into $\rho^2\sigma^2$ to an even power, we have the following result:

THEOREM VIII. *The number $\rho^4 - \sigma^4$, in which ρ and σ are different positive integers, has always an odd prime factor entering into it to an odd power.*

As immediate corollaries of this theorem we have theorems IV and V in §3.

By means of theorem VII we may also prove the following theorem:

THEOREM IX. *The equation $m^4 + n^4 = \alpha^2$ is impossible in integers all of which are different from zero.*

For, if such an equation exists we have a right triangle $(m^2)^2 + (n^2)^2 = \alpha^2$ whose area $\frac{1}{2}m^2n^2$ is twice a square number, which is impossible.

Another proof of theorem V by means of theorem IX is immediate. Theorems IV and IX, taken together, may be stated in the form:

THEOREM X. *In a numerical right triangle $a^2 + b^2 = c^2$, not more than one of the numbers a, b, c is a square.*

PROBLEMS AND QUESTIONS.

B. F. FINKEL, CHAIRMAN OF THE COMMITTEE.

PROBLEMS FOR SOLUTION.

ALGEBRA.

392. Proposed by H. PRIME, Boston, Mass.

The floor and ceiling of a room h feet high are parallel and horizontal. In the middle of the ceiling is a vertical circular tube w feet in diameter extending upward from the ceiling. From the room the longest possible inflexible rod is put up the tube. When the rod is in contact with the opposite sides of the tube and the floor, what is the expression for the tangent of the acute angle made by the rod and the floor in terms of h and w, considering the rod as an air line. To be solved without the calculus. [From *The Maine Farmers' Almanac*, 1913.]

393. Proposed by H. E. TREFETHEN, Waterville, Maine.

Expand $\log [(\sin x)/x]$ in terms of x.

394. Proposed by E. B. ESCOTT, University of Michigan.

Solve the equation $\sin^2 x \sin^2 2x = 5/16$.

395. Proposed by V. M. SPUNAR, Chicago, Ill.

Solve the system of equations $x_1^2x_2 = a_1$; $x_2^2x_3 = a_2$; $x_3^2x_4 = a_3$; $\cdots$ $x_n^2x_1 = a_n$.